DEVELOPMENT OF A RÍO DE LA PLATA WATER LEVEL HEIGHT FORECASTING SYSTEM BASED ON THE MOHID WATER MODELLING TOOL

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CHAPTER SYNOPSIS

Background
In recent years the forecasting models strategy has been developed applying the MOHID water system model in the South Atlantic Ocean and the Río de la Plata. After the initial development of the forecasting system several changes were made in order to improve it. Also, several circulation analyses were performed using the PDT system to improve the understanding of the tidal hydrodynamic in the area.

Results
Several high performance computing tools were applied in order to reduce the computational time using the hardware platform available. Also, several aspects of the South Atlantic hydrodynamic model were evaluated and improved: the study domain, the bathymetry data, the open boundary condition for astronomical tide, and the bottom friction coefficient. Higher changes in the simulated astronomical tide were produced by the improvement in the local bathymetry of Río de la Plata. Moreover, for the coastal control stations located outer the Río de la Plata the most influential parameter was the open boundary condition through the relaxation time.

Conclusions
Using the MOHID hydrodynamic model coupled with the WRF atmospheric model, a forecast water level height system was performed. The system is capable of doing real-time simulations and predicting water levels generated by astronomic and meteorological effects in the Río de la Plata region. This system constitutes a very powerful tool for the management of fluvial-maritime transport.

1 INTRODUCTION

Operational oceanography involves the retrieval, dissemination, and interpretation of measurements obtained in the seas and oceans with the purpose of making a forecast of future conditions [1]. Operational modelling includes simulations of current as well as future conditions. Pre-operational modelling involves similar simulations in terms of initial and boundary conditions, and the specification of forcings, but these are done in hindcast mode. The results presented in this chapter report the progress made in pre-operational modelling for the Río de la Plata fluvial-estuarine system.

Even though the Río de la Plata is very important for Argentina and Uruguay from an economic and environmental point of view, a global management system based on an operational model has not been developed yet. Several problems related with the management of this complex system arose, showing a need to develop an operational system capable of monitoring and predicting hydraulic and environmental variables of interest (water levels, wave climate, currents, salinity, and sediments). In 2008, a research project was carried out in order to develop a pre-operational numerical tool, based on the application of last-generation hydrodynamic and atmospheric numerical models, to support the management of the Río de la Plata – Uruguay River complex system.

The forecasting models strategy has been developed applying the MOHID water system model in the south Atlantic Ocean and the Río de la Plata [2]. The developed system, called PDT system, is capable of predicting sea level variations in the Río de la Plata, and therefore constitutes a numerical tool of great value for the fluvial-maritime navigation and regional environmental management. In order to represent the propagation and generation of astronomic and meteorological waves in the South Atlantic, the model was implemented in a mother domain covering most of the South Atlantic Ocean (hereafter South Atlantic model). Subsequently, a subdomain focused on the Río de la Plata was configured using a nested grid (hereafter Río de la Plata submodel). The results obtained include astronomic and meteorological sea level variations in the Río de la Plata. Comparisons of modelled water levels with data have shown very good qualitative and quantitative agreement [2].

The MOHID modelling system presents an integrated modelling philosophy, not only of processes (physical and biogeochemical), but also of different scales (allowing the use of nested models), and systems (estuaries and watersheds). The MOHID model is considered to be one of the most elaborated models of this type, designed with a reliable and robust framework, and various vertical coordinates.

After the initial development of the forecasting system several changes were made recently in order to improve it. Also, several circulation analyses were performed using the PDT system to improve the understanding of the tidal hydrodynamic in the area. This chapter aims to describe the basics of the PDT system, the improvements made on it, and the analyses performed with the developed tool. At this time the tool is under development and the first stages have been extremely successful. The MOHID water modelling system has several advantages when used in this kind of research projects: the continuous development and improvement, the facility in the exchange of data, and the simplicity of the structured code that allows making modifications with ease.

2 STUDY AREA

Uruguay is located in the east coast of South America between Argentina and Brazil (Figure 1). It has an area of 176,215 km^2 and a population of 3.46 million. To the Southwest, in the border with Argentina, lies the estuary of Río de la Plata. The Río de la Plata is a complex body of water with fluvial-estuarine characteristics located between 34°00' – 36°10' S latitude and 55°00' – 58°10' W longitude. The Río de la Plata basin has the second largest catchment area (3,170,000 km^2) of South America behind the Amazons basin. The Río de la Plata discharges into the South Atlantic Ocean.

The flow dynamics in the Río de la Plata and the ocean front are very complex due to the topographic variation of the river (Figure 1) and the influence of continental flows, astronomical, and meteorological tides coming from the Atlantic Ocean. The Río de la Plata behaves as a micro-tidal estuary, i.e. the river level variations produced by astronomical tides are much lower than those generated by the wind action and oceanic waves. The Paraná and Uruguay rivers provide more than 97% of the continental water inlet with an annual mean flow of 22,000 m^3 s^{-1}. Two main regions can be identified based on the morphology and dynamics of the

Río de la Plata. A shallow area located along the Punta Piedras–Montevideo line separates the inner from the outer region. The inner region has a fluvial regime with bidimensional flow. In the outer region the increase in river width generates complex flow patterns. This outer region is formed by brackish waters of variable salinity according to the tides, winds, and fresh water contributions of the river basin, extending approximately up to Punta Rasa in Argentina and Punta del Este in Uruguay. A more detailed description of the area and data analysis is presented in related studies [3, 4, 5].

3 THE PDT PRE-OPERATIONAL FORECASTING SYSTEM

To describe the circulation of the Río de la Plata, the group of models should consider the atmospheric processes acting over the continental platform and over the adjacent oceanic region. Moreover, the hydrodynamic model should have different spatial resolutions and consider different types of water bodies: on the one hand, the South Atlantic Ocean and its interaction with the atmospheric processes; and on the other hand, the Río de la Plata and lower Uruguay River. This leads to the use of nested domains to describe the system. Therefore, the hydrodynamic model was first implemented on a regional scale domain, over the South Atlantic Ocean, to reproduce the generation and propagation of tidal and meteorological waves. Subsequently, a hydrodynamic submodel with higher resolution was implemented focused on the Río de la Plata and its ocean front. This submodel was nested in the regional scale model. The implemented system was calibrated and validated. Comparisons of modelled water levels with data have shown very good qualitative and quantitative agreement [2]. In this section the main characteristics of the system are briefly presented.

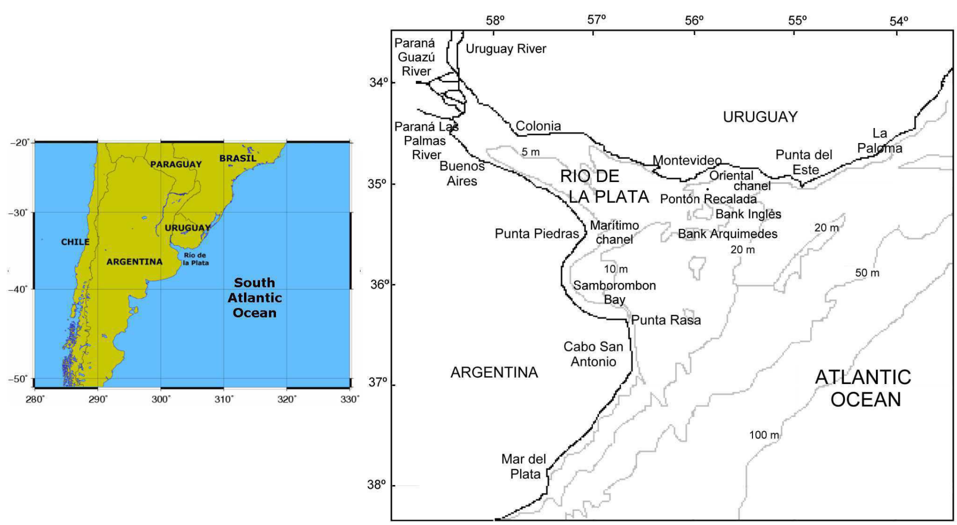

Figure 1. *South Atlantic Ocean and Río de la Plata location.*

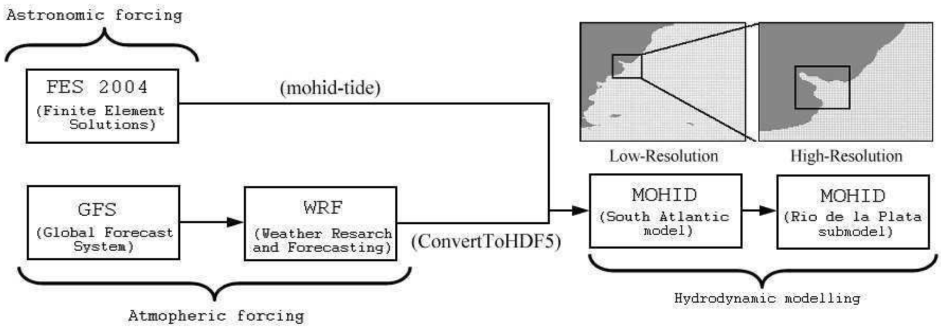

Figure 2. *PDT pre-operational forecasting system scheme.*

3.1 Implementation of the system

The numerical model MOHID was used in this study. The hydrodynamic model makes use of FES2004 (global tidal atlas) solution for tidal forcing and WRF meso-scale model for atmospheric forcing. The WRF model receives the initial and boundary conditions from a global atmospheric model (GFS). Figure 2 presents a scheme of the system of models described above.

The domain of the South Atlantic model is bounded within the interval 22° – 60° S and 20° – 70° W. It includes most of the continental platform and its adjacent oceanic area in order to represent the meteorological effects once forced with the atmospheric model. The horizontal grid of this model was implemented using the latitude-longitude coordinates system, it has 150,131 active nodes and a spatial resolution of 0.1° (11.1 km). Data from the General Bathymetric Chart of the Ocean (GEBCO) with a spatial resolution of 0.06° were used to represent the bathymetry of the area of interest (Figure 3, left panel). The Río de la Plata submodel, unidirectional nested in the South Atlantic model, was implemented using the latitude-longitude coordinates system. Its domain is bounded within the interval 33.76° – 38.09° S and 59.00° – 54.10° W. The horizontal grid has 9,600 active cells and a spatial resolution of 0.033° (3.7 km), one third of the spatial spacing used in the mother model grid. The bathymetry of the Río de la Plata domain and its coastal line was generated using the same information used for the South Atlantic model (Figure 3, right panel).

In the case of the South Atlantic domain tidal forcing is imposed in the open boundary condition. Meteorological forcings (winds and atmospheric pressure from WRF model) are imposed in the surface boundary condition. The Weather Research and Forecasting (WRF) system [6] is designed to be a flexible portable code that is efficient in a massively parallel computing environment. It is suitable for a broad spectrum of applications across scales ranging from meters to thousands of kilometers. The WRF System consists of the following major components: WRF Software Framework (WSF), Advanced Research WRF (ARW) dynamic solver, Standard Initialization package (SI), WRF Variational Data Assimilation (WRF-Var), numerous physics packages contributed by WRF partners and the research community, and several graphics programs and conversion programs for other graphics tools [7].

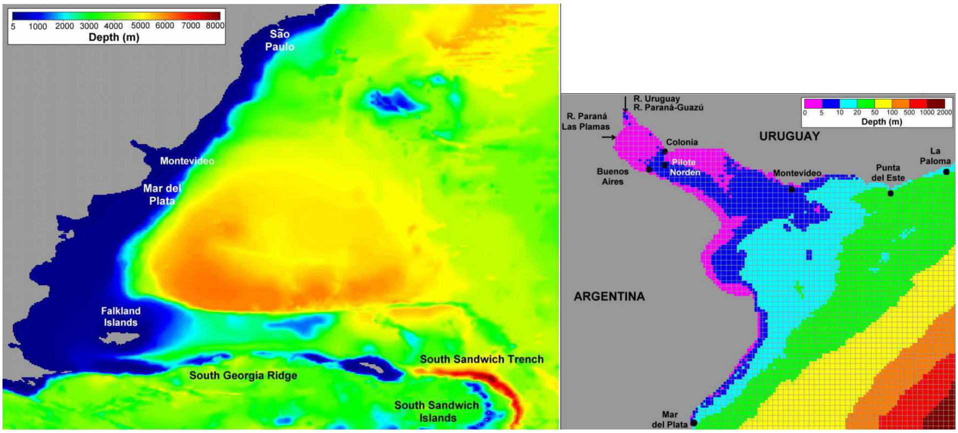

Figure 3. *Bathymetry of the South Atlantic domain (left panel) and bathymetry of the Río de la Plata domain (right panel)*

The WRF was implemented in a domain that ranges approximately from 61.5° W, 76° S to 19° W, 14° S with a spatial resolution of 60 Km. This domain was defined to match the South Atlantic hydrodynamic model domain. The initial and boundary conditions are obtained with the Global Forecast System (GFS) model. Four daily GFS forecasts (produced at 00, 06, 12, and 18 UTC) for up to 180 hours are available at the NCEP website with a temporal resolution of 3 hours and a spatial resolution of 1°. Every WRF atmospheric simulation starts at 00 UTC and results are saved every 3 hours for up to 96 hours. The sea level pressure field and the horizontal components of wind speed at 10m of the sea surface obtained with the atmospheric forecasts are used to force the hydrodynamic model. A conversion files format was applied to the WRF output in order to forcing the MOHID model using the ConvertToHDF5 tool included in the MOHID software package.

3.2 Pre-operational system

The coupling of the hydrodynamic model with the atmospheric forecast model allows the prediction of water levels in the entire Río de la Plata and its ocean front region. At present, the system is still working in a preliminary way. Nevertheless, in order to build a pre-operational system the execution of the models and their interaction were automated. The system runs in Linux due to the better performance of the implemented models in this operating system and because it simplifies the automation process. All the procedure was automated using the shell scripting technique.

The pre-operational system follows the steps described below:

1. Download and store the results of the global atmospheric model GFS;
2. Execution of the atmospheric model WRF using the results of the GFS as initial and boundary conditions;

3. Extraction of the forecast of the atmospheric variables of interest (wind velocity at 10 m and sea level pressure) from the WRF output file;
4. Format conversion and grid interpolation of the atmospheric data into an HDF5 format file;
5. Execution of the MOHID using the forecast obtained with the WRF as the atmospheric forcings;
6. Post-processing of the results.

The pre-operational system was implemented in a computer with LINUX operating system that has two Intel(R) Pentium(R) 4 CPU 2.80 GHz processors, 1024 KB of cache memory and 1 GB of RAM memory. The 4.9 Mohid serial version was used. As an example, a 96 hours forecast was done in approximately 18 hours; 11 hours spent in downloading and storing the GFS data (approx. 890 MB), 2.5 hours spent in the WRF simulations, and 4.5 hours running MOHID. Downloading the atmospheric data from the GFS model is the longest step and could be speeded up by improving the connection.

4 IMPROVEMENTS OF THE SYSTEM

4.1 South Atlantic Hydrodynamic Model

The aim of this analysis was to improve the representation of astronomical tidal sea-surface height variations obtained with the South Atlantic hydrodynamic model described before. To achieve this, the main implementation characteristics of the hydrodynamic model, such as model domain, bathymetry, bottom friction, and open boundary condition, were reviewed. Sensitivity tests to these characteristics allowed us to investigate their individual impact on the results of the model and to obtain an optimal solution as determined by comparisons with several coastal tide gauges. When comparing the results of the model with the sea level series measured at coastal stations, particular attention was given to the most energetic tidal component, the principal lunar semidiurnal M2. A significant improvement in the model tidal representation at the coastal stations of the Río de la Plata was achieved by complementing the GEBCO bathymetric database with local data. Noticeable improvements in tidal representation for most of the coastal stations within the model domain were obtained by tuning the relaxation time (Blumberg and Kantha boundary condition) and the bottom friction coefficient.

For these analyses several simulations of 3 months were performed modifying only one parameter each time. In order to compare the solutions obtained in the simulation, the main tidal harmonics from the water level series were computed in several coastal control stations (Figure 4). The main tidal harmonic in the area, the semidiurnal M2 component, was specially controlled. Measured water level series are available at the Río de la Plata coastal control stations (Figure 4, right panel) and theoretical tidal amplitude and phase of the main harmonic M2 are available in the Argentinean continental platform (Figure 4, left panel).

The RMSE (root mean square) normalized by the observed amplitude was used to compare the M2 measured amplitude and phase with the M2 modelled amplitude and phase for each control station:

$$RMSE = \frac{\sqrt{\frac{1}{2}\left(H_M^2 - 2H_M H_T \cos\left(G_M - G_T\right) + H_T^2\right)}}{H_T} \quad (1)$$

Where H_M and G_M are the amplitude and phase of the M2 harmonic calculated from the model, and H_T and G_T are the amplitude and phase of the theoretical M2 harmonic calculated from the measurements. Moreover, in order to compare the results in the rest of the domain, the isophase and isoamplitude maps were computed for each simulation. For doing this, a harmonic analysis was performed to the hourly water level series computed by the model in each cell of the domain. The amplitude and phase for each tidal component was obtained in each cell and finally, using the contour function in MATLAB Software, the maps were obtained. The harmonic constants were calculated from the simulated series using the t_tide tool [8].

4.1.1 Model domain revision

The South Atlantic domain used in the PDT system was revised. A sensitivity test using the original domain and other two different domains was performed (Figure 5). For the sensitivity test, a simulation of 1 year with astronomical forcing was made using each domain. To compare the results obtained with the domains a control point net was defined in the intersection zone of the three domains. The hourly water level series at each point of this net was used in the comparison. Firstly, the Principal Component (EOF) Analysis was applied to the water level series. Secondly, parameters such as the RMSE value were used to compare the results. The sensitivity test showed that the main aspects of the tidal propagation in the Río de la Plata are well represented by the domain number 2 (Figure 5). This domain is smaller than the original domain, the number 1 in Figure 5, so the consequence is a reduction in the computational cost.

4.1.2 Bathymetry improvement

In this stage the bathymetry generation procedure was revised and corrected (Figure 6). The GEBCO data used for the original bathymetry was merged with better local data for the Río de la Plata zone. The GEBCO data of less than 50 m at the area between 33.5^o S and 38.5^o S were eliminated. In this way the GEBCO data with more incertitude were replaced by better quality bathymetry data from the Río de la Plata zone. The transition zone between the two types of data was controlled in order to avoid discontinuities. Moreover, a filter using the MOHID tool FilterBathymetry was applied. The filter applies an averaging over the cells depth smoothing the differences in depth between the adjacent cells. The new bathymetry was finally constructed with a combination of a filtered version and a non filtered version; in the outer Río de la Plata zone the filtered version with a radius (number of adjacent cell) equal to 5 was used, and in the inner Río de la Plata zone the data by the non filtered version was used. In the Figure 6 the original and the new bathymetry of the Río de la Plata zone are compared. With the new bathymetry the simulated astronomical tide was improved; the normalized RMSE obtained with all control stations decreases from 37.8% to 21.7% while the normalized RMSE obtained only with the Río de la Plata control stations decreases from 39.2% to 16.4%.

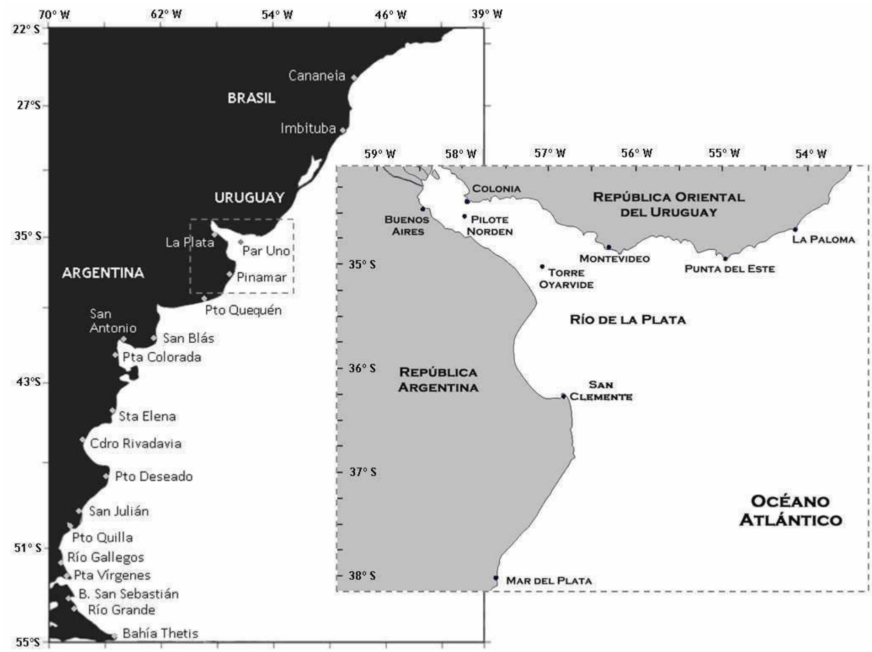

Figure 4. *Location of the coastal control stations used for amplitude and phase comparison of the M2 main harmonic.*

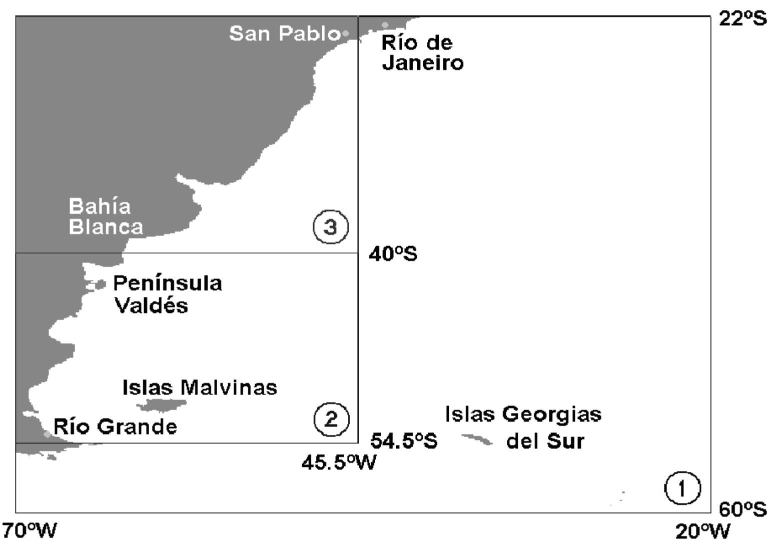

Figure 5. *South Atlantic domains used in the analysis.*

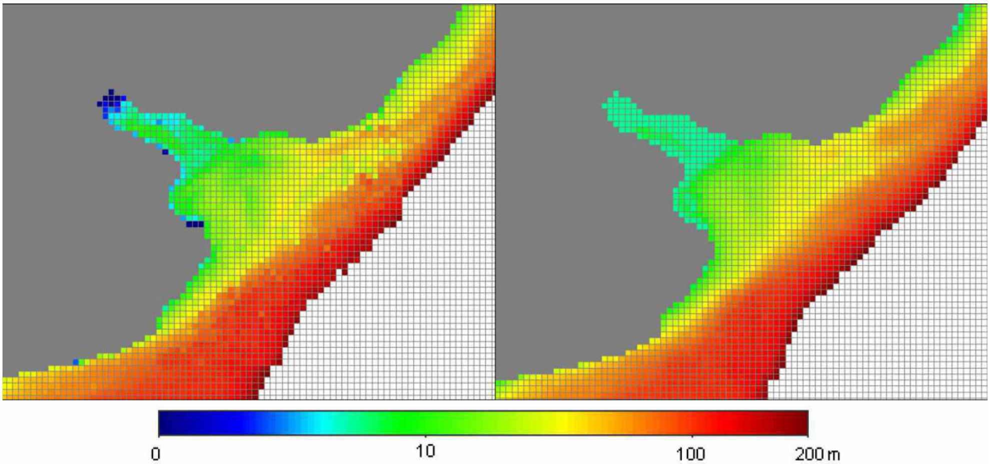

Figure 6. *Original bathymetry (left panel) and the improved bathymetry (right panel) for the Río de la Plata zone.*

4.1.3 Open boundary condition revision

The open boundary condition in the Atlantic South domain was reviewed. In the astronomical tidal forcing simulations the open boundary condition is used to impose the tidal waves. The tidal waves are calculated from the amplitudes and phases to the 13 harmonic components: M2, N2, S2, K2, 2N2, O1, Q1, K1, P1, Mf, Mm, Mtm, MSqm. This information is extracted from the FES2004 global atlas by the mohid-tide tool, for every other cell throughout the entire open boundary of the domain. Also the MOHID model allows the imposition of a radiation condition in the open boundary. In this stage of the improvement of the original condition a radiation boundary condition was established. Some authors claim that the best radiation open boundary condition for the water level elevation is the Flather condition [9]. Nevertheless, the Flather proposed radiation condition needs a reference solution for the water level elevation and the barotropic velocity at the boundary. In this case only the water level elevation is known, so the Blumberg and Kantha [10] radiation condition is a good alternative, for which we must define the relaxation time.

Following the recommendations for the relaxation time, 3 different configurations were evaluated (a, b, and c). For the boundary cells with major depths (greater than 3000 m) the relaxation time was 100 s (a), 200 s (b), and 300 s (c); whereas for the boundary cells with minor depths (lower than 200 m) the relaxation time was about 1000 s (a), 1500 s (b), and 2000 s (c); following the relationship between them 1:10. For the boundary cells with depths between 200 m and 3000 m a transition relaxation time was defined linearly proportional to the relaxation time of the shallow and deep cells defined above.

The obtained results show that the boundary relaxation time has an influence over the tidal propagation in the Argentinean coastal stations but not in the Río de la Plata zone. The RMSE calculated for the Río de la Plata zone did not change along the three configurations analyzed. The best solution (with the minor normalized RMSE value) was obtained with the configuration of relaxation times 100 s and 1000 s. Moreover the obtained amplitude and phase maps for each simulation were very similar.

4.1.4 Adjustment of the bottom roughness coefficient

The bottom roughness coefficient for the Atlantic Ocean domain was adjusted using the bottom friction formulation presented in Equation [2]; where n is the Manning number, g is the gravity acceleration, and h is the depth. Three different configurations were defined differing in the Manning number: 0.005, 0.010, and 0.015. Three different simulations were performed with the three defined Manning numbers imposed to the entire domain. The best results were obtained with the lower Manning number, equal to 0.005. Moreover, it was observed that the isoamplitude and isophases maps were not so sensible to the bottom roughness coefficient.

$$C_f = \frac{2gn^2}{h^{1/3}} \tag{2}$$

5 APPLYING THE HIGH PERFORMANCE TOOLS STRATEGY

The hardware platform employed to carry out the experiments is a multicore machine with eight cores. In order to evaluate the developed tool, the MOHID model and its related tools were installed on a Linux operated system.

Firstly, we compiled the source code using the Intel Fortran compiler and fine tuned the binary generated using the optimization flags. Additionally, two different parallel programming tools available on MOHID were studied, OpenMP and MPI. The MPI standard was used to improve the performance of the resolution of the hydrodynamic model following a nested domain strategy, each nested domain was computed by one MPI process using the pipeline paradigm. On the other hand, the OpenMP API was used to reduce the runtime needed to compute each domain following the shared memory paradigm.

Thus, both HPC techniques allowed us to compute large domains with high precision grids on a reasonable runtime, and this improvement enabled the use of the simulation tool in an operational mode.

6 SYSTEM APPLICATIONS

6.1 Study of the main astronomical waves

The improved South Atlantic hydrodynamic model was used to study the tidal astronomical waves in the zone. In Figure 7 the amplitude and phase maps for the semidiurnal M2 tidal component for the entire domain (left panel) and for the Río de la Plata (right panel) are presented. The improved model shows the main characteristic of the M2 tidal component in the zone. The RMSE is 24% when considering all coastal control station, and it decreases to 19.1% when considering only the coastal control stations located at Río de la Plata.

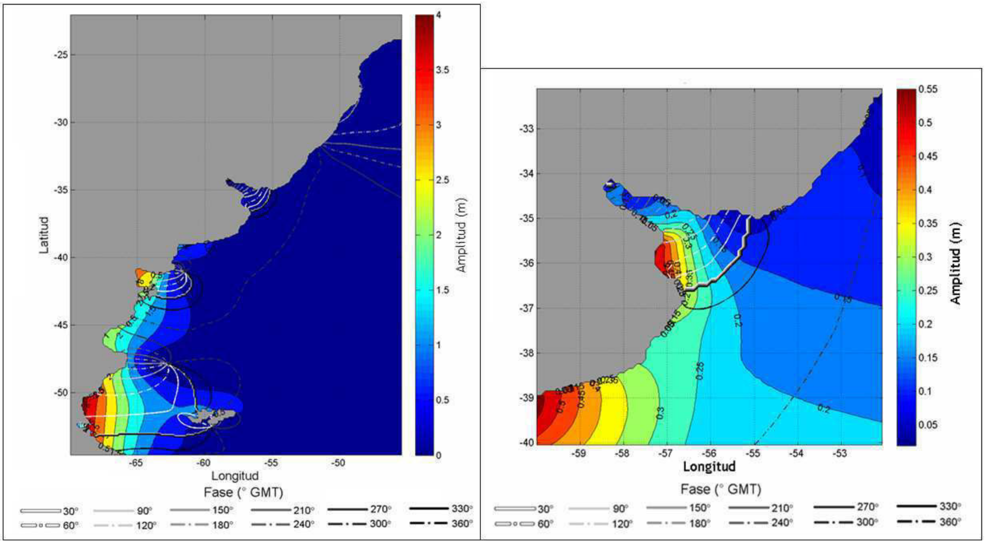

Figure 7. *Amplitude and phase maps for the M2 constituent obtained with the improved model for the entire domain (left panel) and a detail of the Río de la Plata zone (right panel).*

6.2 Determination of the meteorological tide generation

The meteorological tide generation and propagation characteristics of the Río de la Plata are examined in this section using the original South Atlantic Río de la Plata modelling system described before. Also, the evaluation of the relative importance of the remote and the local forcings was performed based on numerical experiments with some simplification.

First, a simulation without meteorological forcing was made to obtain a pure astronomical solution, and then this solution was subtracted to the solutions including meteorological forcings to remove the astronomical variability. Two simulations were made to observe the generation zones of the meteorological tide events observed in Río de la Plata using wind and sea level pressure (SLP) as forcings. The first one includes wind and SLP forcings in both the regional domain and Río de la Plata sub domain (called Rwp_Swp), and it aims to evaluate the generation of meteorological tide events in the father domain which are associated to the remote forcing. The second one includes wind as SLP forcing only in the Río de la Plata domain (called R_Swp), so it aims to evaluate the generation of meteorological tide events only by the action of the meteorological forcings in a local spatial scale. The simulated period was June – July, 2007.

The water level series obtained from the complete simulation Rwp_Swp were compared with the water level series measured at several points in the Río de la Plata, checking the accuracy of the model reproducing the meteorological events. Nevertheless, the meteorological water level series obtained with the simulation R_Swp, only with atmospheric forcing in the local model, did not reproduce the whole meteorological measured signal thus showing the relevance of the remote forcing.

To determined where the meteorological tide events are generated, correlation maps between the water level series at one location inside the Río de la Plata (at Pilote Norden, see location in Figure 1 right panel), and the water level series for each cell of the domain were calculated. This procedure was repeated for different temporal lags, allowing us to see where was the signal observed at Pilote Norden at previous instants. The time series extracted from the domain cells were processed with the Matlab software. Figure 8 shows the correlations maps for various temporal lags, correlations greater than 0.8 have been highlighted by filling the contour map. It should be noted that maximum correlations take place approximately at Mar del Plata for lag 17 hours. For greater lags the region of maximum correlation is located at higher latitudes, showing that 40 hours before the observation of the meteorological tide events at Pilote Norden the signal was situated approximately at 45^{o} S.

These results show where the events were generated. A similar analysis calculating correlation maps was made between the observed sea level heights and the Reanalysis wind fields to identify the forcing conditions that generate the events. As a result, maximum correlations were obtained between the filtered sea level height series at Pilote Norden and the southwest component of the wind.

7 CONCLUSIONS

Using the MOHID hydrodynamic model coupled with the WRF atmospheric model, a forecast water level height system was performed. The system is capable of doing real-time simulations and predicting water levels generated by astronomic and meteorological effects in the Río de la Plata region. This system constitutes a very powerful tool for the management of fluvial-maritime transport. Nevertheless, the relevance of the system for the management of the area requires continuous development and improvement of the system.

The modelling system consists of a group of integrated models. Although the area of interest is the Río de la Plata region, the system of models needs to be implemented in a bigger domain that includes a large area of the South Atlantic Ocean and then, using the nesting approach, the model can focus on the Río de la Plata region. The atmospheric forcings introduced in the hydrodynamic model are obtained with the WRF model while the astronomic forcings are obtained from the FES2004 global tidal atlas. The WRF is initialized with the GFS global model forecasts.

The original system is now going through an improvement stage. Several high performance computing tools were applied in order to reduce the computational time using the hardware platform available. Also, several aspects of the South Atlantic hydrodynamic model were evaluated and improved: the study domain, the bathymetry data, the open boundary condition for astronomical tide, and the bottom friction coefficient. The improvement in the local bathymetry in the Río de la Plata produced the bigger changes in the simulated astronomical tide. Moreover, for the coastal control stations located outer the Río de la Plata the most influential parameter was the open boundary condition through the relaxation time. The least influential parameter over the simulated M2 tidal component characteristic was the Manning number evolved in the bottom roughness coefficient formulation.

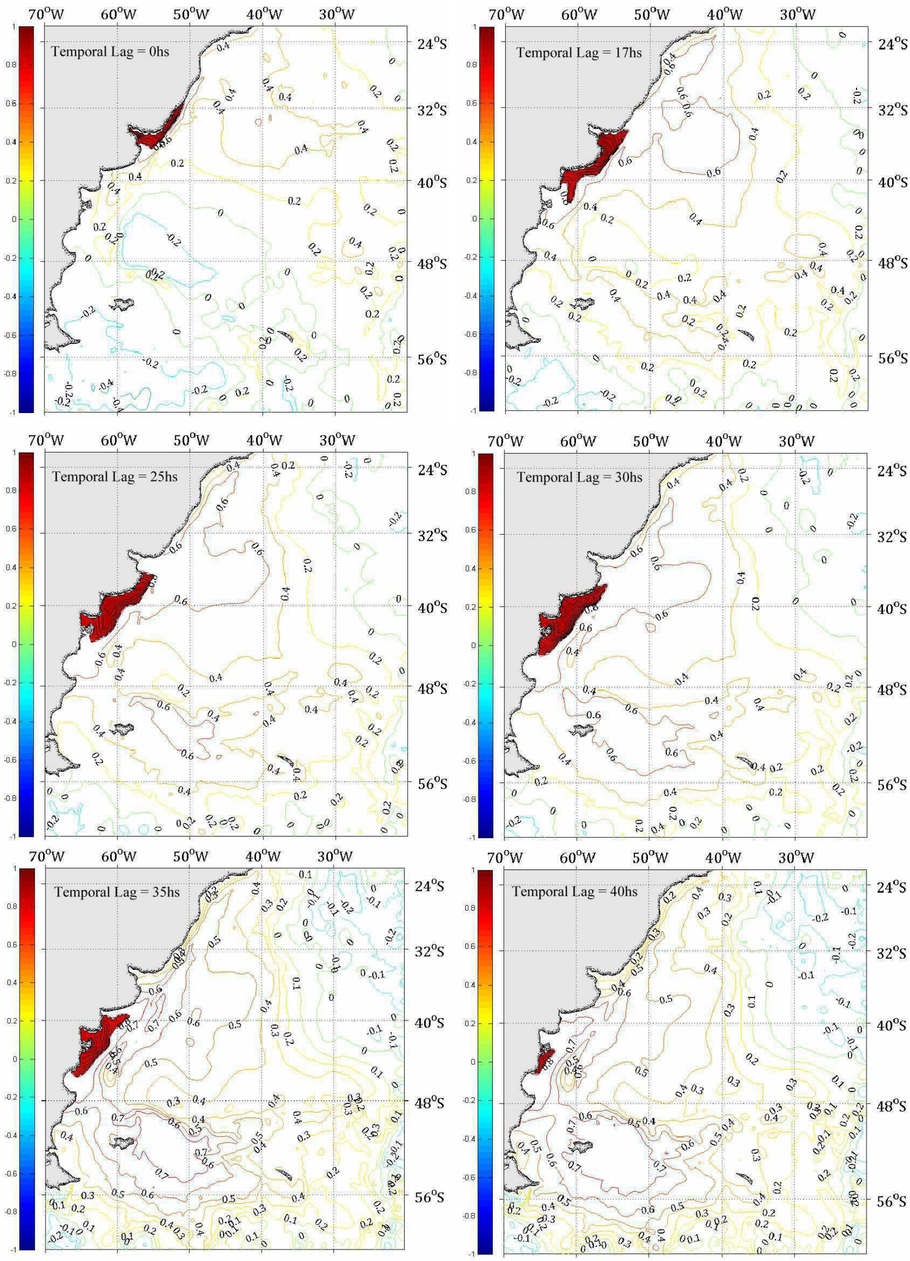

Figure 8. *Correlation maps among the modelled meteorological tide at Pilote Norden and the rest of the domain for different temporal lags.*

REFERENCES

1. Prandle, D., Operational Oceanography - A view ahead, Coastal Eng. 41 (2000) 253-259.
2. Santoro, P.; Fernández, M.; Fossati, M.; Cazes, G.; Terra, R.; Piedra-Cueva, I. Pre-operational forecasting of sea level height for the Río de la Plata. Applied Mathematical Modelling 35 (2011) 2462-2478
3. Piedra Cueva, I. Fossati, M., Residual currents and corridor of flow in the Río de la Plata, Applied Mathematical Modelling 31 – 3 (2007) 564-577.
4. Fossati, M., Piedra-Cueva, I., Numerical modelling of residual flow and salinity in the Río de la Plata, Appl. Math. Modell. 32 (2008) 1066–1086.
5. Fossati, M.; Bellón, D; Lorenzo, E.; Piedra-Cueva, I. Currents measurements in the coast of Montevideo, Uruguay. RCEM 2009 Proceedings. River, Coastal and Estuarine Morphodynamics. Santa Fe, Argentina, september 2009.
6. Michalakes, J., J. Dudhia, D. Gill, T. Henderson, J. Klemp, W. Skamarock, W. Wang, The Weather Research and Forecast Model: Software Architecture and Performance, Proceedings of the Eleventh ECMWF Workshop on the Use of High Performance Computing in Meteorology. Eds. Walter Zwieflhofer and George Mozdzynski. World Scientific (2005) 156 – 168.
7. Wang, W., Barker , D., Bruyère , C., Dudhia, J., Gill. D., Michalakes, J., Advanced Research WRF (ARW) Version 2.1 modelling system User's Guide (2005).
8. Pawlowicz, R.; Beardsley, B.; Lentz, S. Classical tidal harmonic analysis including error estimates in MATLAB using T TIDE. Computers & Geosciences 28, 2002, pp. 929–937.
9. Flather, A. (1976). A tidal model of the northwest European continental shelf. Mem. Soc. R. Sci. Liege, 10 (6), pp. 141–164.
10. Blumberg, A. & Kantha, F. (1985). Open boundary condition for circulation models. Journal of Hydraulic Engineering, ASCE, Vol. 111, pp. 237-255.